Oops! Unreasonable effectiveness of mathematics in natural sciences and engineering

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“Pure,” “applied” mathematics, and math teaching are inseparable.
We live in the exciting time of great unification!
Communication between AMS, SIAM, and MAA is required.
Doing (good) applied mathematics is harder!
“Somebody once said that philosophy is the misuse of a terminology which was invented just for this purpose. In the same vein, I would say that mathematics is the science of skillful operations with concepts and rules invented just for this purpose.”

Eugene Wigner, Physics Nobel Prize Laureate
Ancient Greeks discovered Mathematics as a fascinating game of intellectual pleasure. Like in art, music, chess, architecture, there is beauty in mathematics.
A beautiful proof
A beautiful formula Euler’s formula $e^{i\pi} = -1$.

an ugly formula
Mathematics, rightly viewed, possesses not only truth, but supreme beauty, a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as in poetry. Bertrand Russell
Other similar games, which we do NOT call sciences

Chess (Go, checkers, card games, sudoku ...): a game with strict rules that were invented just for intellectual exercise (= fun)

It does not matter, how and of what the board and pieces are made
What is different in mathematics?

It works **magic** in physics, engineering, chemistry, biology, finances, you name it.

*There are things which seem incredible to most men who have not studied Mathematics.*

Archimedes of Syracuse (287-212 B. C. E)

*Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of the world.*

Roger Bacon (1214-1294)

*Philosophy is written in this grand book – I mean the universe – which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language in which it is written. It is written in the language of mathematics, ... , without which it is humanly impossible to understand a single word of it; without these, one is wandering about in a dark labyrinth.*

Galileo Galilei (1564 - 1642)
why?

We wish someone knew the answer!

*The most incomprehensible thing about the universe is that it is comprehensible. .... How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?*  
Albert Einstein

...the enormous usefulness of mathematics in the natural sciences is something bordering on the *mysterious* and that there is no rational explanation for it. ... The *miracle* of the appropriateness of the language of mathematics for the formulation of the laws of physics is *a wonderful gift which we neither understand nor deserve.*  
Eugene Wigner
Math Is...

Universal

Oops! Unreasonable effectiveness of mathematics in natural science  Seeing invisible: mathematics of medical imaging
Active discussion by physicists, philosophers, and rare mathematicians

Mario Livio, Is God a Mathematician?, Simon & Schuster 2010
Max Tegmark, Our Mathematical Universe: My Quest for the Ultimate Nature of Reality, Knopf 2014
The Value of Science: Essential Writings of Henri Poincare, Random House, 2012
Who is a mathematician?

A mad hatter!, who builds a ladder to nowhere, just for the heck of it

... and suddenly discovers that it leads to the top of a previously unsurmountable mountain!
Ancient Greeks studied (just for their beauty) planar sections of cones:

Two millennia later, Kepler discovered that planet’s trajectories are ellipses. Galileo discovered that trajectories of projectiles are parabolas. This led Newton to the gravity law. And the contemporary science began!
The favorite since ancient times. First serious applications in 20th century to coding.
Riemann introduced in 19th century the famous $\zeta$-function $\zeta(z)$. Properties of its zeros, i.e. points where $\zeta(z) = 0$ are crucial for number theory.
Millennium prize $1$ mil is out there.
Recent discovery: these zeros behave like the spectra of complex quantum mechanical systems (e.g., heavy atoms).
WHY???
We wish we knew!
Solving equations

Quadratic formula for the roots of quadratic equation $ax^2 + bx + c = 0$ was known probably since Babylonians (although not to all our school students).
Formulas for the roots of cubic and fourth order equations – obtained two millennia later, in Italy during the Renaissance times.
Fifth order - no luck!!! Who cares about solving fifth order equations anyway?
Oops! Such formula does not exist! Proven By Abel, Galois, and Ruffini in the beginning of 19th century.
The techniques developed laid the foundation of the Group Theory.
For a hundred years, it was an example of a useless creation of mad hatters, till quantum mechanics came along.
Nowadays textbooks on quantum physics and chemistry (and even on medical imaging) include basics of group theory.
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Seeing invisible: mathematics of medical imaging
We are lucky to live in the world where frequencies stabilize (some times).
Toss of a true coin: frequencies of heads and tails stabilize in the long run to 0.5 each.

WHY???
Matrix theory, invented in 19th century, was not even known to most physicists and considered being “another esoteric math thing.”

Till, in 1920s, Max Born noticed that what W. Heisenberg was doing resembled the matrix algebra. This has lead to the contemporary quantum mechanics.
Knot theory - another example.
Why does this happen?

How can the result of a long chain of thought, according to the rules invented by us and far removed from reality, ever be useful?
We wish we knew!

Why do we choose in our brain not absolutely useless constructions, but those that eventually invariably become useful (although this might take hundreds of years)?
We wish we knew!

Why does the nature agree with our conclusions?
We wish we knew!
Mathematics can handle only VERY simplified models. Simplifying real life problems, one needs to have specific knowledge in order not to throw away the most important things. All models have some limiting assumptions they are build upon. Know those and do not believe a formula as the scripture! Experimental mathematics: educated guesses, conjectures, numerical evidence, etc. have value! Applied mathematics: very often one cannot rigorously justify the procedure. This is bad and someone should try to overcome this, but this is not a reason to neglect the procedure if it seems to work.
And now some imaging!

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2.2 Å resolution cryo-EM structure of β-galactosidase
Thank you for your patience!
Enjoy the use and beauty of math!